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by

I. M. Yenukashvili

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В в	*В в*	V, v	Т т	*Т т*	T, t
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Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α α		Nu	Ν ν	
Beta	Β β		Xi	Ξ ξ	
Gamma	Γ γ		Omicron	Ο ο	
Delta	Δ δ		Pi	Π π	
Epsilon	Ε ε ϵ		Rho	Ρ ρ ϱ	
Zeta	Ζ ζ		Sigma	Σ σ ς	
Eta	Η η		Tau	Τ τ	
Theta	Θ θ ϑ		Upsilon	Τ υ	
Iota	Ι ι		Phi	Φ φ ϕ	
Kappa	Κ ϰ κ		Chi	Χ χ	
Lambda	Λ λ		Psi	Ψ ψ	
Mu	Μ μ		Omega	Ω ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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INTEGRAL EQUATION OF THE KINETIC THEORY OF THE COAGULATION OF CLOUD PARTICLES.

I. M. Yenukashvili.

Page 470.

Investigations in the theory of transfer of cloud particles in considerable degree are bonded with the development of the methods of the solution to kinetic equations. In this respect by very promising is presented writing of the kinetic equation of coagulation in the form of integral equation. In particular this is exhibited during the solution to the boundary-value problems of the kinetics of coagulation. This integral equation, including boundary conditions, in very compact form contains entire physics of the process of the transfer of cloud particles and can render/show more convenient than integrodifferential equation. Let us note that the integral kinetic equations successfully are utilized in the theory of transfer of neutrons [1], in aerodynamics of rarefied gases [2, 3].

The transport problem of cloud particles - this the problem of many particles and with study of the formation of the spectrum of

cloud particles in space and in time must be considered their absolute motions, mutual collisions and merging/coalescences. Simultaneously with this the value of the distribution function of cloud particles in point with radius-vector r (x, y, z) they must be bonded not only with the values of distribution function in the points, close toward the end of vector r, but also with values in all points of cloud, including interfaces.

Let n (v, r, t) be the distribution function of cloud particles according to volumes v at the moment of time t about point in space with radius-vector r, so that value

$$dn = n(v, r, t)\,d\Omega\,dv \tag{1}$$

is for the moment of time t the mathematical expectation of the number of cloud particles, which have radius-vector and range r, r + dr and volume in the range v, v + dv; dΩ = dx dy dz - elementary three-dimensional/space common/general/total, that adjoins toward the end of radius-vector r.

Distribution function n (v, r, t) in question makes sense of the density of the mathematical expectation of the number of cloud particles; however, together with this it simultaneously has also the sense of the density of distribution of the probability of the detection of one particle in the four-dimensional volume dvdΩ.

Let us isolate in cloud the small volume element dΩ and determine in this volume about point with radius-vector r for the arbitrary moment of time t, the number of cloud particles with volume in the range v, v + dv, i.e., let us determine dn = n (v, r, t) dΩ dv.

Page 471.

It is obvious, this number will be determined by the intensity of the emergence (generation) of cloud particles by the volume in the range v, v + dv in all cloud (including interfaces) at the different moments of time, which precede the moment of time t in question, and also by the probability of the admission of the emergent cloud particles into the volume dΩ in question without collision with other cloud particles.

Let Π (v, r, t, τ) be a probability of free motion in the time interval (τ, t) of this cloud particle, which at the moment of time τ is located in point with radius-vector r- (t-τ) with and at the moment of time t proves to be at the point of space with radius vector r. Here c is speed of the motion of the cloud particle, which has volume v, which depends on v, and also on the velocity of the

motion of air masses in cloud. The generalization of the physical derivation of the system of integral equations of the kinetics of the coagulation of cloud particles in the case of the accelerated motion of cloud particles presents no difficulties [4]. Special examination requires the case of the motion of cloud particles in the turbulent flow of air masses.

Let us examine certain cloud particle at certain moment of time t. Let it is locate in point with radius-vector r and has volume v and speed c. It is possible to assert that at certain moment of time τ < t it experienced the last/latter collision and that it as the cloud particle, possessing volume v and speed c, it was born in point with radius-vector r- (t-τ) c at the moment of time τ.

Further, let Φ (v, r, t) be a function of generations, so that value

$$dn_1 = \Phi(v, r, t)\, d\Omega\, dv\, dt \qquad (2)$$

gives the mathematical expectation of the number of cloud particles, which were being born in the elementary volume of space dΩ about point with radius vector r for a period of time dt and which have volume in the range v, v + dv.

Let us express function n (v, r, t) through the function of

generations and the probability of free motion. At the moment of time $t > t_0$ (t_0 - the initial moment) the mathematical expectation of the number of cloud particles with a volume of v, $v + dv$ in the volume element of space $d\Omega$ is composed of two parts.

$$n(v, r, t)\,dv\,d\Omega = dn_0 + dn_2, \qquad (3)$$

where dn_0 is a mathematical expectation of the number of cloud particles with volume in the range v, $v + dv$, which reached in $d\Omega$ from the initial state without collisions; dn_2 is a mathematical expectation of the number of cloud particles with a volume of v, $v + dv$, which appear in cloud in time interval from t_0 to t at the different moments of time τ at the different points of space with radius-vector $r-(t-\tau)c$ and which without collisions reach $d\Omega$ up to the moment of time t.

It is obvious that

$$dn_0 = n[v, r-(t-t_0)c, t_0]\,\Pi(r, v, t, t_0)\,dv\,d\Omega. \qquad (4)$$

Page 472.

Then, according to determination (2), value

$$\Phi[v, r-(t-\tau)c, \tau]\,d\Omega\,dv\,d\tau \qquad (5)$$

will be equal to the mathematical expectation of the number of cloud particles, which are born within volume dΩ about point with radius-vector r- (t-τ) c for a period of time dτ and have a volume in the range v, v + dv. From this quantity of cloud particles to element of volume dΩ about the points with radius-vector r will r ach only the those, that for the extent/elongation of time (τ, t) do not experience collisions. Therefore, if we expression (5) multiply by the probability of free motion Π (r, v, τ, t) and to integrate over alternating/variable τ from t_0 to t we will obtain the total quantity of cloud particles with a volume of v, v + dv, which appe r in cloud in time interval (t_0, t) and the moment of time t enterelement of volume dΩ about point with radius-vector r. Thus,

$$dn_2 = d\Omega\, dv \int_{t_0}^{t} \Phi\,[v,\ \mathbf{r} - (t - \tau)\mathbf{c},\ \tau] \cdot \Pi(\mathbf{r},\ v,\ \tau,\ t)\, d\tau. \qquad (6)$$

Special examination requires the case, when occurs the emergence of cloud particles on interface, for example at the level of condensation. For those regions of the clouds, into which can enter cloud particles, which arose on interface in time interval (t_0, t), in relationship/ratio (3) one should write the additional term, which considers a quantity of cloud particles with volume in the range v, v + dv, which appear on interface at the appropriate moment of time and which without collision reach dΩ about point with radius-vector r up to the moment of time t. In this case, it goes without saying, the boundary function of the generations is assumed known.

Substituting (4) and (6) in (3), we obtain

$$n(v, \mathbf{r}, t) = n[v, \mathbf{r} - (t - t_0)\mathbf{c}, t_0] \Pi(\mathbf{r}, v, t, t_0) + \\ + \int_{t_0}^{t} \Phi[v, \mathbf{r} - (t - \tau)\mathbf{c}, \tau] \Pi(\mathbf{r}, v, \tau, t) d\tau. \quad (7)$$

For obtaining the closed system of integral equations it is necessary to express the probability of free action and the function of generation by distribution function.

Let ΔQ_i be a probability of the random event, which consists of the fact that during time interval $\Delta\tau_i$ examined cloud particle is adjacent to another cloud particle.

Page 473.

Then we have

$$\Delta Q_i = \Delta\tau_i \int_0^{\infty} n[u, \mathbf{r} + \tau_i \mathbf{c}, t + \tau_i] \sigma(v, u) du,$$

where

$$\sigma(v, u) = \pi E \left(\frac{3}{4\pi}\right)^{2/3} (v^{1/3} + u^{1/3})^2 |\mathbf{c}(v) - \mathbf{c}(u)|$$

- the effective volume of the collisions of the cloud particles, which have volumes v and u; E is a coefficient of capture

With small $\Delta\tau_i$ the probability of free motion for the cloud particle in question during time interval $\Delta\tau_i$ will be equal to

$$P_i = 1 - \Delta Q_i = e^{-\Delta Q_i}.$$

Then the probability of the free motion of the cloud particle in question for a period of time $T = \sum_{i=1}^{n} \Delta\tau_i$ will be

$$P = \prod_{i=1}^{n} P_i = \exp\left\{-\sum_{i=1}^{n} \Delta Q_i\right\} = \exp\left\{-\sum_{i=1}^{n}\left[\int_0^\infty n(u, \mathbf{r} + \tau_i \mathbf{c}, t + \tau_i)\,\sigma(v, u)\,du\right]\Delta\tau_i\right\}.$$

Hence, replacing summing by integration, we obtain

$$\Pi(v, \mathbf{r}, \tau, t) = \exp\left\{-\int_\tau^t \left[\int_0^\infty \sigma(v, u)\, n(u, \mathbf{r} - (t - q)\mathbf{c}, q)\,du\right] dq\right\}. \tag{8}$$

The last/latter relationship/ratio gives communication/connection

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between the probability of free motion and the distribution function of cloud particles.

To the generation of cloud particles with a volume of v in the volume of space dΩ for time dt give the collisions of cloud particles from volumes v-u, u, with this u it can take values from 0 to v. Therefore the collision frequency, summed up in terms of all possible values of u, gives interesting us the mathematical expect tion of the number of generations of cloud particles with a volume of v, thus

$$\Phi(v, r, t)=\frac{1}{2}\int_0^v n(v-u, r, t)n(u, r, t)\sigma(v-u, u)du. \quad (9)$$

Expression σ (v, u) reflects mechanics and the statistics of the reaction between the pair of cloud particles.

equations (7), (8), (9) are the system of the integral equations which characterize the kinetics of the coagulation of cloud particles. The structure of these equations is such, that easily it is possible to produce the exception/elimination of functions Π, Φ and to obtain one integral equation for the distribution function of cloud particles.

Page 474.

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Substituting (8), (9) in (7), we have

$$n(v, r, t) = n[v, r-(t-t_0)c, t_0] \exp\left\{-\int_{t_0}^{t}\left[\int_0^\infty \sigma(v, u) n(u, r - (t-q)c, q) du\right] dq\right\} + \int_{t_0}^{t} \exp\left\{-\int_{\tau}^{t}\left[\int_0^\infty \sigma(v, u) n(u, r - (t-q)c, q) du\right] dq\right\}\left[\frac{1}{2}\int_0^v n(v-u, r-(t-\tau)c, \tau) n(u, r-(t-\tau)c, \tau) \sigma(v-u, u) du\right] d\tau. \quad (10)$$

Let us use the operation

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (c \cdot \underline{\Delta})$$

to both parts of equation (7); taking into account the relationship/ratios:

$$\frac{d}{dt}\{n[v, r-(t-t_0 c, t_0]\} = 0,$$

$$\Pi(r, v, t) = \Delta,$$

$$\frac{d\Pi}{dt} = -\Pi\int_0^\infty \sigma(v, u) n(u, r, t) du,$$

we obtain

$$\frac{dn}{dt}=\Phi(v,\ \mathbf{r},\ t)-\int_0^\infty \sigma(v,\ u)n(u,\ \mathbf{r},\ t)du\left\{\int_{t_0}^{t}\Phi[v,\ \mathbf{r}-(t-\tau)\mathbf{c},\ \tau]\times\right.$$
$$\left.\times\Pi(\mathbf{r},\ v,\ \tau,\ t)d\tau+n[v,\ \mathbf{r}-(t-t_0)\mathbf{c},\ t_0]\Pi(\mathbf{r},\ v,\ t,\ t_0)\right\}.$$

If, according to (7), expression in curly braces in the last/latter relationship/ratio is replaced through n (v, r, t), and function Φ (v, r, t) - with expression (9), then we will obtain the known integrodiff equation of the kinetic theory of the coagulation of the cloud particles

$$\frac{dn}{dt}=-n(v,\ \mathbf{r},\ t)\int_0^\infty \sigma(v,\ u)n(u,\ \mathbf{r},\ t)du+$$
$$+\frac{1}{2}\int_0^v \sigma(v-u,\ u)n(v-u,\ \mathbf{r},\ t)n(u,\ \mathbf{r},\ t)du.$$

Page 975.

As the numerical methods of the approximate solution of the system of integral equations (7), (8), (9) it is possible to utilize a method successive approximation, an iterative method, and also a method of torque/moments [5, 6]. Accepting as zero approximation either the initial distribution or any analytical solution of the simplified problem of the kinetics of the coagulation of cloud particles, we compute the function of generating Φ_0 and the probability of free motion Π_0. Substituting Φ_0 and Π_0 in equation (7), we obtain the first approximation of distribution function n_1. Further we compute Φ_1, Π_1, n_2. The following approach/approximations are constructed analogously.

In conclusion let us note that the proposed above form of the notation of the physical derivation of the system of the integral equations of the kinetics of the coagulation of cloud particles can be successfully used also in the examination of the kinetics of the coagulation of dispersed particles and, in particular aerosol particles.

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